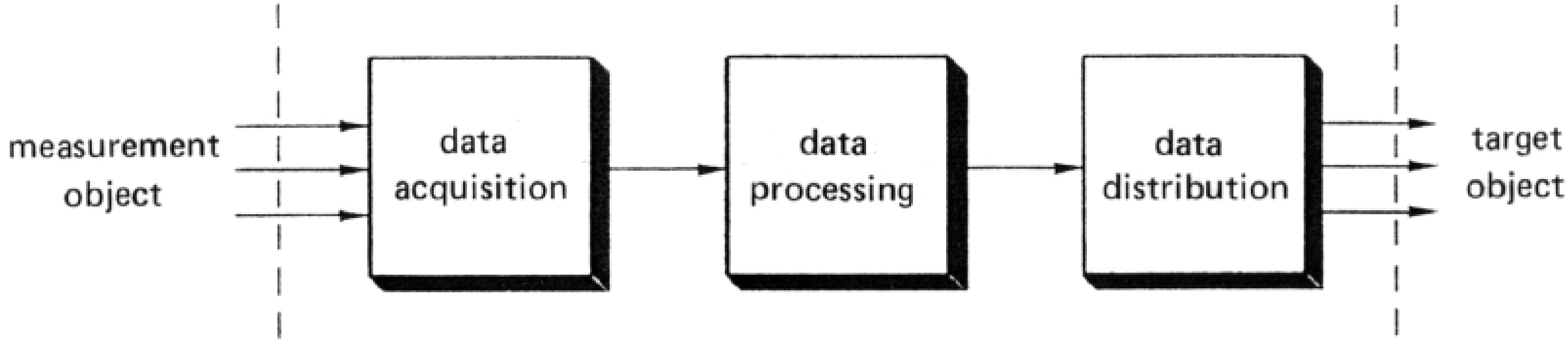
Electronic Instrumentation for Measurement

Introduction

Contents

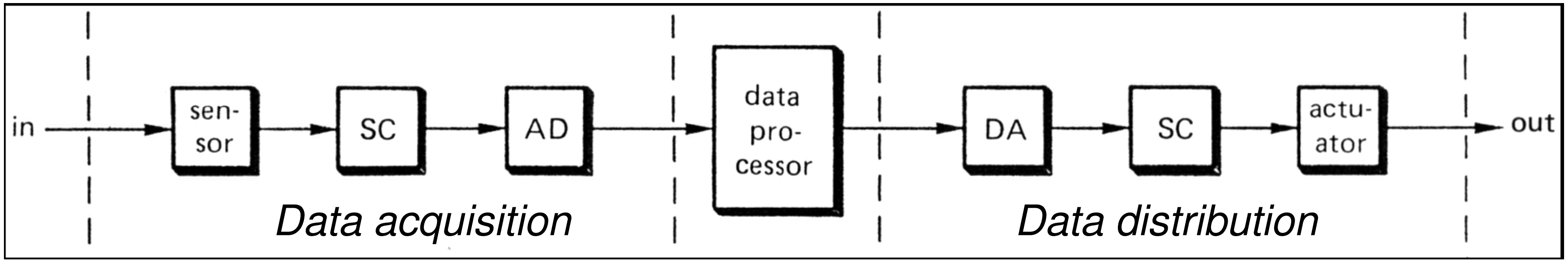
- **Measurement** = process of comparing the unknown quantity with an accepted standard quantity (u.m.);
- **Measuring system aims:**
  - to obtain information about a physical process;
  - to find appropriate ways of presenting that information to an observer or to other automatic systems;
- **Measuring system functions:**
  - *data acquisition* - acquiring information about the object to be measured and converting into electrical measurement data;
  - *data processing* – selecting, processing or manipulating measured data (usually math operations);
  - *data distribution* - supplying of measured data to the target object (display for human, comm. interface for machine).

Measuring system functions:

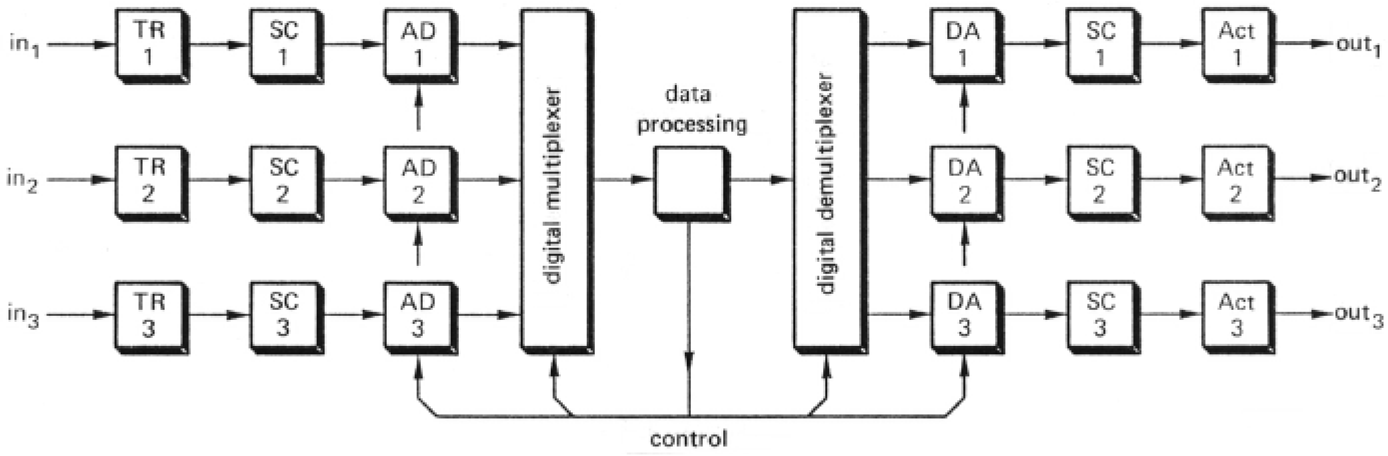


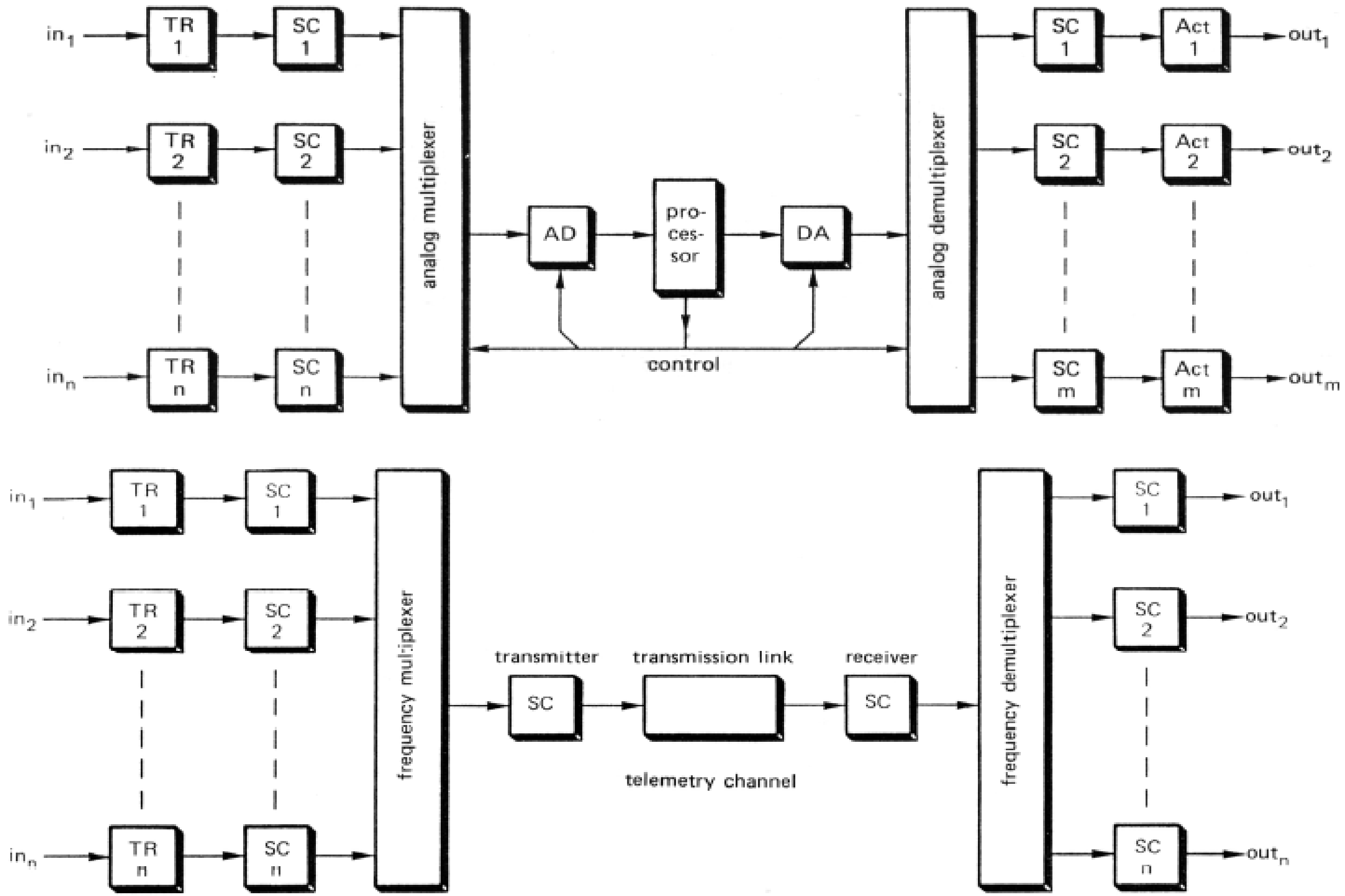
- *Data acquisition*:
  - Sensor or transducer - produces an electrical analog signal (obtain electrical information, $u(t)$, $i(t)$ in case of non-electrical data measurement – bijective function);
  - Signal conditioning: amplification, filtering, modulation, demodulation, non-linear operations of electrical signal;
  - AD-converter: sample & hold, quantization, binary encoder;

- *Data distribution*
  - DA converter (optional);
  - Signal conditioning (optional): the DAC output signal is adapted to actuator input: antialiasing filtering, amplification, filtering, non-linear operations;
  - Actuator (effector) - transforms the electrical signal into the desired non-electric form. Type of actuator functions: indicating (on display), storing (memory, CD, printer, etc), controlling (valve, heating element, electrical dive, etc);



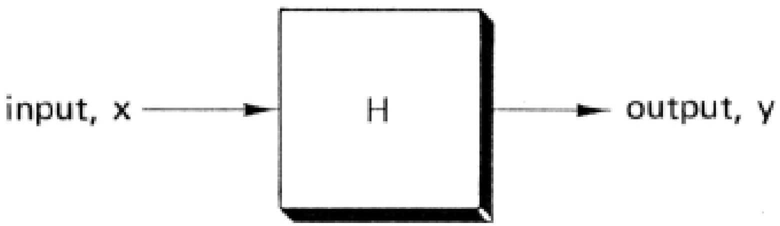
- Multi-channel measuring system
  - central processor and digital multiplexer (time division) - fast data processing, slow ADC, DAC ;
  - centralized processor and AD and DA-converters and analog multiplexer (time division) - fast data processing, ADC, DAC ;
  - system with frequency multiplexing (frequency division) - telemetry;

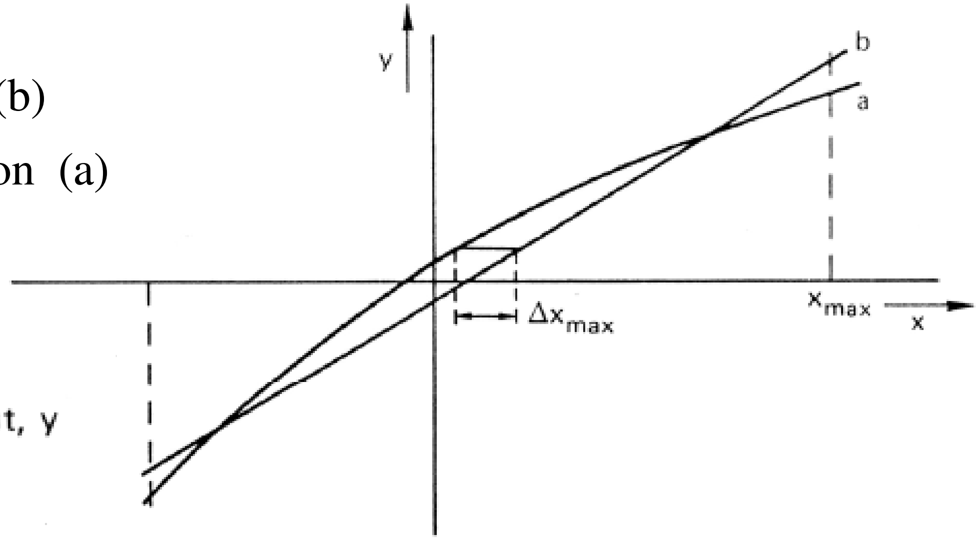



in1
in2
inn
TR 1
TR 2
TR n
SC 1
SC 2
SC n
analog multiplexer
AD
pro-ces-sor
DA
control
analog demultiplexer
Act 1
Act 2
Act m
SC m
out1
out2
outm
frequency multiplexer
transmitter
SC
transmission link
telemetry channel
receiver
frequency demultiplexer
outn

- **Measuring system specifications**
  - *measurement range* (0 – 100V, 0-2A, etc) - the input range between the specified maximum value (Full-scale FS) and minimum value (usually 0) where the system can be used for measurement;
  - *resolution* - the smallest change of input quantity - output detectable;
  - *sensitivity* - ratio between the output value variation (*y*) to the input variation (*x*) that causes that output change (linear / nonlinear function: saturation, clipping, dead zone);

$$S = \frac{\Delta y}{\Delta x} = \begin{cases} const & \text{for linear function (b)} \\ S(x) & \text{fo non-linear function (a)} \end{cases}$$





- **Measuring system specifications (cont'd)**
  - *bandwidth* – the input frequency span between frequencies (f_l-f_u) where the system output has dropped to half from the corect value;
  - *accuracy* - how precise the measured value is it (compared to the real value) - opposed to inaccuracy;
  - *input impedance* (1MΩ||27pF);
  - environmental operating range:
    - *supply voltage* (220V- 50Hz, 110V-60Hz, etc) ;
    - the *environmental conditions:* operational temperature (*–10°C to 40°C*), storage temperature (*-20°C to 85°C*), humidity (10% to 95%), altitude (0m to 6000m), etc.
    - *other parameters* : load (>20Ω);
  - *reliability of the system* (failure rate $\lambda(t)$ or the mean-time-to-failure MTTF);

Accuracy of measurement

Classical way–*error of measurement* (instant or maximum)

- absolute error $e_X = X_m - X_{ad}$ where $\begin{cases} X_m & \text{measured value} \\ X_{ad} & \text{true value} \end{cases}$
- relative error $\varepsilon_X = \dfrac{e_X}{X_{ad}} \cong \dfrac{X_m - X_{ad}}{X_m}$
- accuracy $A_x = 1 - \varepsilon_X$
- error propagation $e_Y = \sum_{k=1}^{N} \left| \dfrac{\partial F(X_1, X_2, \ldots X_N)}{\partial X_k} \cdot e_{X_k} \right|$

$$\varepsilon_Y = \frac{1}{F(X_1, X_2, \ldots X_N)} \cdot \sum_{k=1}^{N} \left| \frac{\partial F(X_1, X_2, \ldots X_N)}{\partial X_k} \cdot X_k \cdot \varepsilon_{X_k} \right|$$

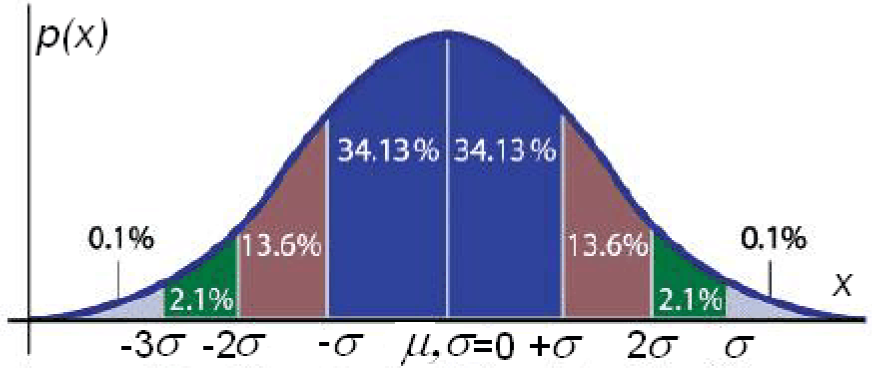
Accuracy of measurement

Statistical way – Standard uncertainty ~ **standard deviation** of variable ***x***

- probability density function (pdf) $p_X(x)$
- probability $\Pr(x_1 \le X \le x_2) = \int_{x_1}^{x_2} p_X(x)\,\mathrm{d}x$
- statistical mean $\bar{X} = \mu = \int_{-\infty}^{+\infty} x \cdot p_X(x)\,\mathrm{d}x$
- statistical variance $\sigma^2 = \overline{(X-\mu)^2} = \int_{-\infty}^{+\infty} (x-\mu)^2 \cdot p_X(x)\,\mathrm{d}x$
- standard deviation $\sigma = \sqrt{\sigma^2} = \sqrt{\overline{(X-\mu)^2}}$

Gauss distribution (normal)

$$p_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2} \right)$$



Standard deviation: σ

Interval of x

$(\mu \mp 0.6745 \cdot \sigma) \Leftrightarrow P(x) = 50.00\%$

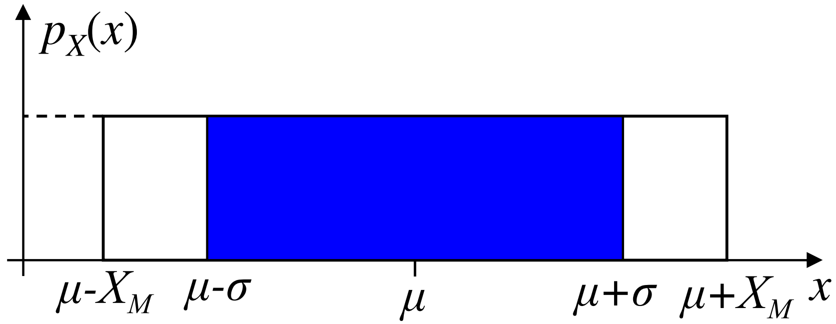
$(\mu - \sigma, \mu + \sigma) \Leftrightarrow P(x) = 68.28\%$

$(\mu - 2\sigma, \mu + 2\sigma) \Leftrightarrow P(x) = 94.46\%$

$(\mu - 3\sigma, \mu + 3\sigma) \Leftrightarrow P(x) = 99.72\%$

Uniform distribution

$$p_X(x) = \begin{cases} \dfrac{1}{2 \cdot X_M}, & x \in [\mu - X_M, \mu + X_M] \\ 0 & \text{otherwise} \end{cases}$$



Standard deviation $\sigma = \frac{X_M}{\sqrt{3}}$

$$x \in (\mu - \sigma, \mu + \sigma) \Leftrightarrow P(x) = 58\%$$

Practical measurement accuracy

Evaluation from *N* samples (ergodic process supposition)

- **Mean** $$\bar{X} = \mu = \frac{1}{N}\sum_{n=1}^{N} x_n$$
- **error** in the n-*th* measurement $$e_{X\_n} = x_n - \bar{X} \qquad \varepsilon_{X\_n} = \frac{e_{X\_n}}{\bar{X}}$$
- **deviation** of in the n-*th* measurement $$e_{X\_n} = x_n - \bar{X}$$
- **Average deviation** $$D_{X_N} = \frac{1}{N}\sum_{k=1}^{N}\left(x_k - \bar{X}\right)$$
- **precision** of the n-*th* measurement $$P_{X\_n} = 1 - \frac{\left|x_n - \bar{X}\right|}{\bar{X}}$$

Practical measurement accuracy

- **Standard deviation** (N>30) $$\sigma_X = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(x_k - \mu\right)^2}$$
- **Standard deviation** (N<30) $$\sigma_X = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N}\left(x_k - \mu\right)^2}$$
- **Uncertainties propagation** $$Y = F\left(X_1, X_2, \ldots, X_K\right)$$

$$\sigma_Y = \sqrt{\sum_{k=1}^{K}\left(\frac{\partial F\left(X_1, X_2, \ldots, X_K\right)}{\partial X_k}\right)^2 \cdot \sigma_{X_k}^2}$$

Least mean squares linear fitting

Simplest case: one measurand is linear function of single independent variable

For N samples of measured variable

$$X = \{x_1, x_2, ..., x_N\}$$

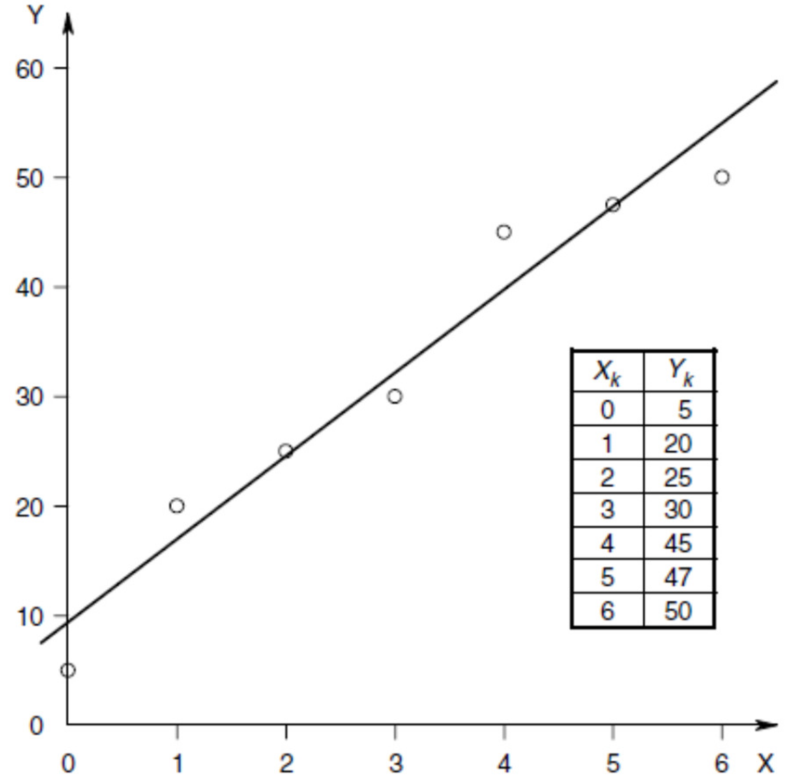
and N samples of measured function

$$Y = \{y_1, y_2, ..., y_N\}$$

Linear fit → determine optimum *m, b*:

$$\{y_n, x_n\} : \quad y_n = m \cdot x_n + b$$

for every *n*



Least mean squares linear fitting

Minimize mean square error (MSE)

$$MSE = \sigma_y^2 = \frac{1}{N}\sum_{n=1}^{N}\left(\left(m \cdot x_n + b\right) - y_n\right)^2$$

$$MSE = 1/N \sum_{n=1}^{N}\left(m^2 x_n^2 + b^2 + 2mbx_n + y_n^2 - 2y_n\left(m \cdot x_n + b\right)\right)$$

Set derivates equal to zero

$$\begin{cases} \dfrac{\partial \sigma_y^2}{\partial m} = 0 \\ \dfrac{\partial \sigma_y^2}{\partial b} = 0 \end{cases} \Rightarrow \begin{cases} m \cdot S_{xx} + b \cdot S_x = S_{xy} \\ m \cdot S_x + b \cdot N = S_y \end{cases}$$

where

$$S_{xx} = \sum_{n=1}^{N} x_n \cdot x_n \quad ; \quad S_x = \sum_{n=1}^{N} x_n$$

$$S_{xy} = \sum_{n=1}^{N} x_n \cdot y_n \quad ; \quad S_y = \sum_{n=1}^{N} y_n \quad ; \quad \sigma_x^2 = \frac{1}{N} S_{xx} - \frac{1}{N^2} S_x{}^2$$

Least mean squares linear fitting

- **Solutions:** $b = \frac{1}{\sigma_x^2}\left(S_x \cdot S_{xy} - S_y \cdot S_{xx}\right) \quad ; \quad m = \frac{1}{\sigma_x^2}\left(S_x \cdot S_y - N \cdot S_{xy}\right)$
- $R_{xy}(0)$ – cross correlogram function evaluated at t =0

$$R_{xy}(0) = \frac{1}{N}\sum_{n=1}^{N} x_n y_n = \frac{1}{N} S_{xy}$$

- r - correlation coefficient for the LMS fit

$$r \triangleq \frac{R_{xy}(0) - \overline{X}\,\overline{Y}}{\sigma_X \sigma_Y} = \frac{\frac{1}{N} S_{xy} - \frac{1}{N^2} S_x S_y}{\sigma_X \sigma_Y} \qquad 0 \le r \le 1$$

r = 1 - perfect fit

- r^2 - coefficient of determination of the fit

- SI (System International Unit)
  - International Standard
  - ……
- Fundamentals

Length	meter	m
Mass	kilometer	kg
Time	second	s
Temperature	degree Kelvin	°K
Luminous Intensity	candela	cd
Electric Current	ampere	A

- Derived

Electromotive Force	volt	V
Quantity of Charge	coulomb	C
Electrical Resistance	ohm	Ω
Capacitance	farad	F
Inductance	henry	H

- Supplementary bibliography

  - S. Rabinovich, Measurement Errors and Uncertainties - Theory and Practice 3rd ed. – 2005;
  - P.P.L. Regtien, Electronic instrumentation, second edition – 2005;