Fiber Optic Communications

Ch 4. Optical amplifiers

Contents

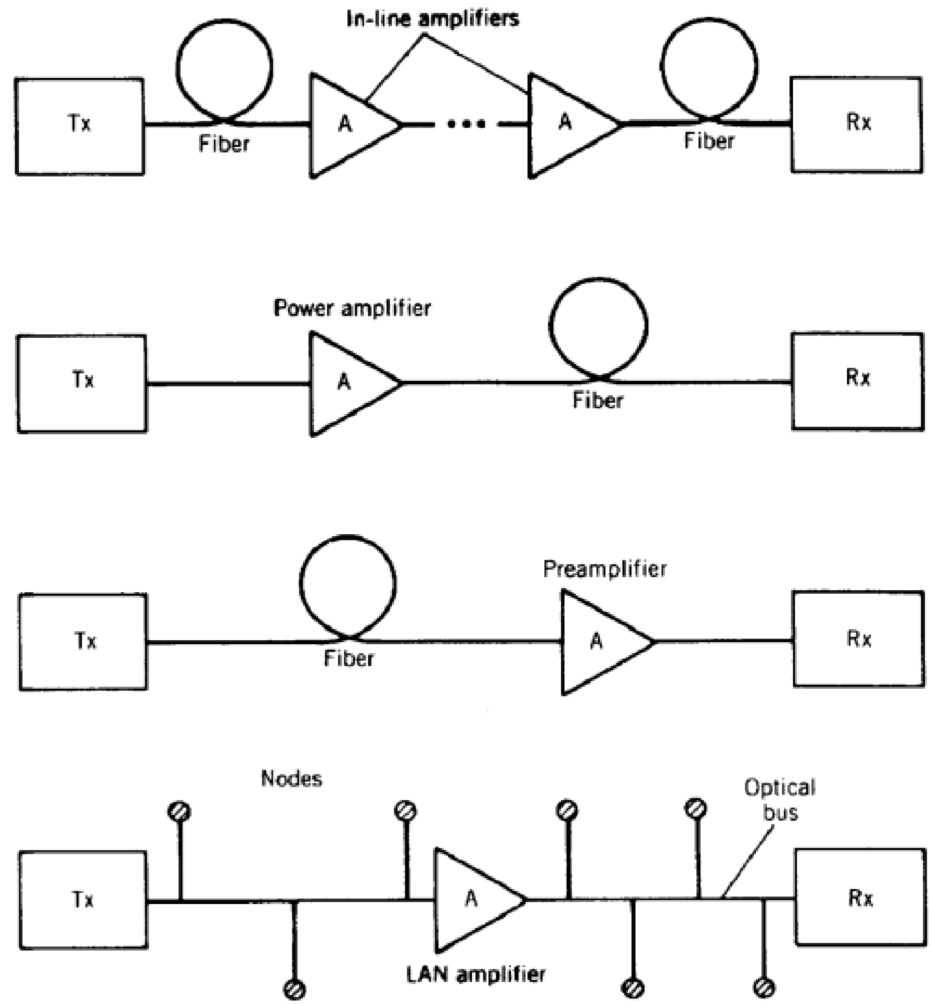
Optical amplifier types

- Doped fiber amplifiers use excitation of ions in the host fiber
  - The ***erbium-doped fiber amplifier*** (EDFA) is most common
    - Optically pumped by laser light at higher energy (shorter wavelength)
- ***Raman*** and ***Brillouin amplifiers*** use nonlinear processes to transfer energy from the pump wave to the signal
  - Vibrations (phonons) in the silica glass are involved in the process
- ***Parametric amplifiers*** use a nonlinear process (FWM) to transfer energy from the pump wave to the signal
- ***Semiconductor optical amplifiers*** (SOA) are electrically pumped
  - Also called semiconductor laser amplifier (SLA)
  - In principle, a semiconductor laser biased below threshold

Amplifier applications

Four main applications:

- **In-line:** Compensates for transmission losses
- **Power amp:** Increases the transmitter output power
- **Pre-amp:** Enhances the sensitivity of the receiver
- (**LAN amp:** Compensates for coupling losses in a network)



General concepts

- Amplification can be ***lumped*** or ***distributed***
  - An EDFA is lumped
    - Gain occurs in a short piece of fiber
  - Raman amplifiers are often distributed
    - Gain occurs within the transmission fiber itself
- An EDFA relies on ***stimulated emission***
  - A stimulated transition to a lower energy level ⇒ emission of a photon
  - Energy is ”pumped” into the medium to induce ***population inversion***
    - Without population inversion, absorption will dominate
  - Even in the absence of an input photon, ***spontaneous emission*** occurs
    - Will add noise in optical amplifiers
- In a Raman amplifier, power is transferred from a pump wave
  - Pump has shorter wavelength (~1.45 µm for 1.55 µm signal)
  - Pumping can be done in forward or backward direction (or both)
    - Backward pumping minimizes transfer of pump intensity noise

Benefits and requirements of Optical Amplif

Benefits:

- Eliminates the need for optoelectronic regenerators in loss-limited systems
- Can improve the receiver sensitivity
- Can increase the transmitted power
- Can be used at all bit rates and for all modulation formats
- Can amplify many WDM channels simultaneously

Requirements:

- An ideal amplifier has
  - High gain, high output power, and high efficiency
  - Large gain bandwidth
  - No polarization sensitivity
  - Low noise
  - No crosstalk between WDM channels
  - Ability to amplify broadband analog and digital signals (kHz – 100’s GHz)
  - low coupling losses to optical fibers

Lumped versus distributed amplification

- Lumped amplification
  - The optical power decreases as $P_{out} = P_{in} \exp(-\alpha z)$
  - With amplifier spacing L_A, the gain is adjusted to $G = \exp(\alpha L_A)$
    - Typical spacing is 30–100 km
  - The spacing must not necessarily be uniform
- Distributed amplification
  - Denoting the gain by $g_0(z)$, we get

$$\frac{dp(z)}{dz} = [g_0(z) - \alpha]p(z)$$

  - Ideally $g_0(z) = \alpha$, but the pump power is not constant ⇒ gain decreases with distance from pump source
  - Condition for compensation over distance L_A is

$$\int_0^{L_A} g_0(z)\,dz = \alpha L_A$$

  - L_A is then known as ***pump-station spacing***

Bidirectional pumping scheme

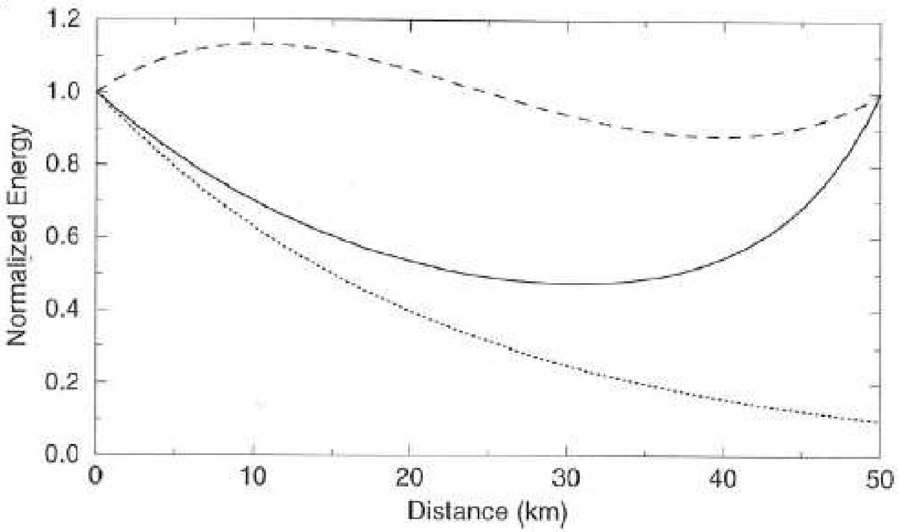
- In bidirectional pumping, the gain is $g(z) = g_1 \exp(-\alpha_p z) + g_2 \exp[-\alpha_p (L_A - z)]$
  - α_p is the loss at the pump wavelength
  - g_1 and g_2 are constants proportional to the launched pump power
- Assuming equal pump powers and that initial and final signal power = 1

$$p(z) = \exp\left[\alpha L_A \left(\frac{\sinh[\alpha_p (z - L_A/2) + \sinh(\alpha_p L_A/2)}{2\sinh(\alpha_p L_A/2)} \right) - \alpha z \right]$$

- Assuming backward pumping ($g_1 = 0$)

$$p(z) = \exp\left[\alpha L_A \left(\frac{\exp(\alpha_p z) - 1}{\exp(\alpha_p L_A) - 1} \right) - \alpha z \right]$$

- Figure shows backwards and bidirectional Raman amplification
  - EDFA case shown for comparison
  - Raman evens out power fluctuation



Gain in a pumped medium

- We consider
  - A two-level system, i.e., there are two different energy levels
  - Population inversion is obtained with either optical or electrical pumping
- The gain coefficient, g [m^{-1}], depends on the frequency ω and the intensity P of the signal being amplified
- The gain has a ***Lorentzian shape***

$$g(\omega)=\frac{g_0}{1+(\omega-\omega_0)^2T_2^2+P/P_{sat}}$$

  - g_0 is the peak gain coefficient determined by the amount of pumping
  - ω_0 and T_2 are material parameters
- The ***saturation power*** is denoted by P_{sat}
- When P = P_{sat}, we have

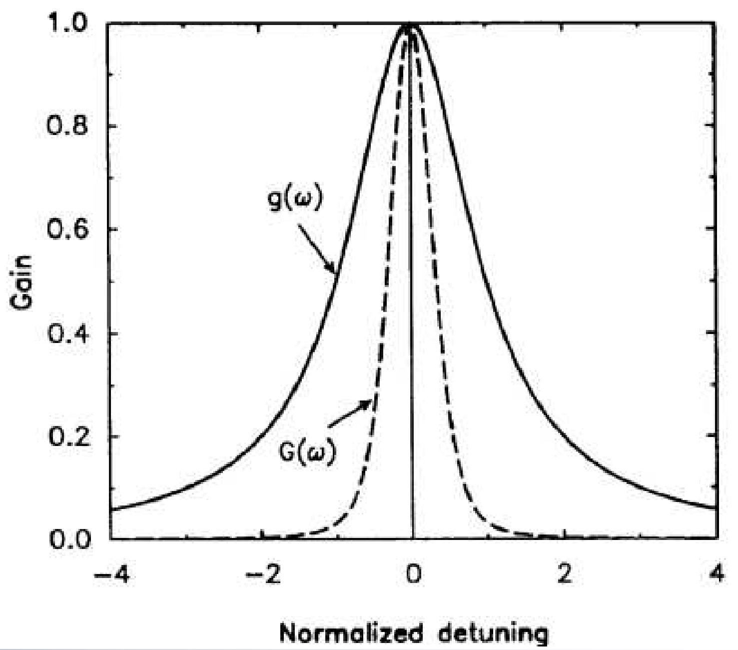
$$g(\omega_0)=\tfrac{1}{2}g_0$$

Unsaturated gain

- When P << P_{sat}, we can neglect the saturation term
- The FWHM bandwidth of the spectrum is

$$\Delta\nu_g = \Delta\omega_g/(2\pi) = 1/(\pi T_2)$$

- The ***amplifier bandwidth*** is of more interest
  - Use
$$\frac{dP(z)}{dz} = g(\omega)P(z)$$
    to get the power gain
$$P(z) = P(0)\exp[g(\omega)z]$$
  - The amplifier bandwidth is obtained from
$$G(\omega) = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P(L)}{P(0)} = \exp[g(\omega)L]$$
    and is found to be
$$\Delta\nu_a = \Delta\nu_g\sqrt{\frac{\ln 2}{g_0L - \ln 2}}$$



Saturated gain

- Study the gain at the gain peak

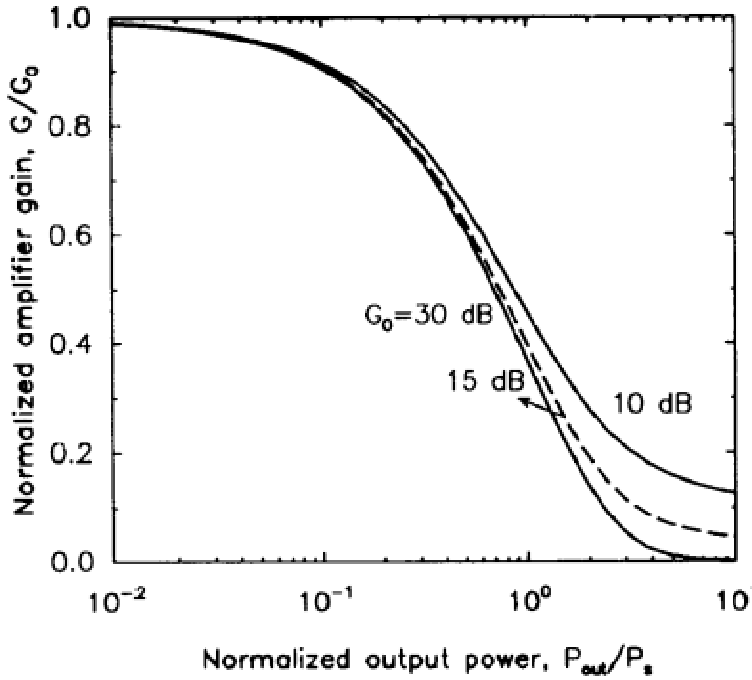
$$\frac{dP(z)}{dz} = \frac{g_0 P(z)}{1 + P(z)/P_{sat}}$$

- Integrating using $P(0) = P_{in}$ and $P(L) = P_{out}$, we get

$$G = \frac{P_{out}}{P_{in}} = G_0 e^{-\frac{G-1}{G}\frac{P_{out}}{P_{sat}}} \qquad G_0 = e^{g_0 L}$$

  - G_0 is the small signal gain
- The ***output saturation power*** is defined as the power when $G = G_0/2$
  - Independent of G_0 for large gain

$$P_{out}^{sat} = \frac{G_0 \ln 2}{G_0 - 2} P_{sat} \approx \{G_0 >> 1\} \approx P_{sat} \ln 2 \approx 0.7 P_{sat}$$



Erbium-doped fiber amplifiers (EDFA)

- The silica fiber acts as a host for erbium ions
  - Erbium can provide gain close to 1.55 μm
  - Optically pumped to an excited state to obtain gain

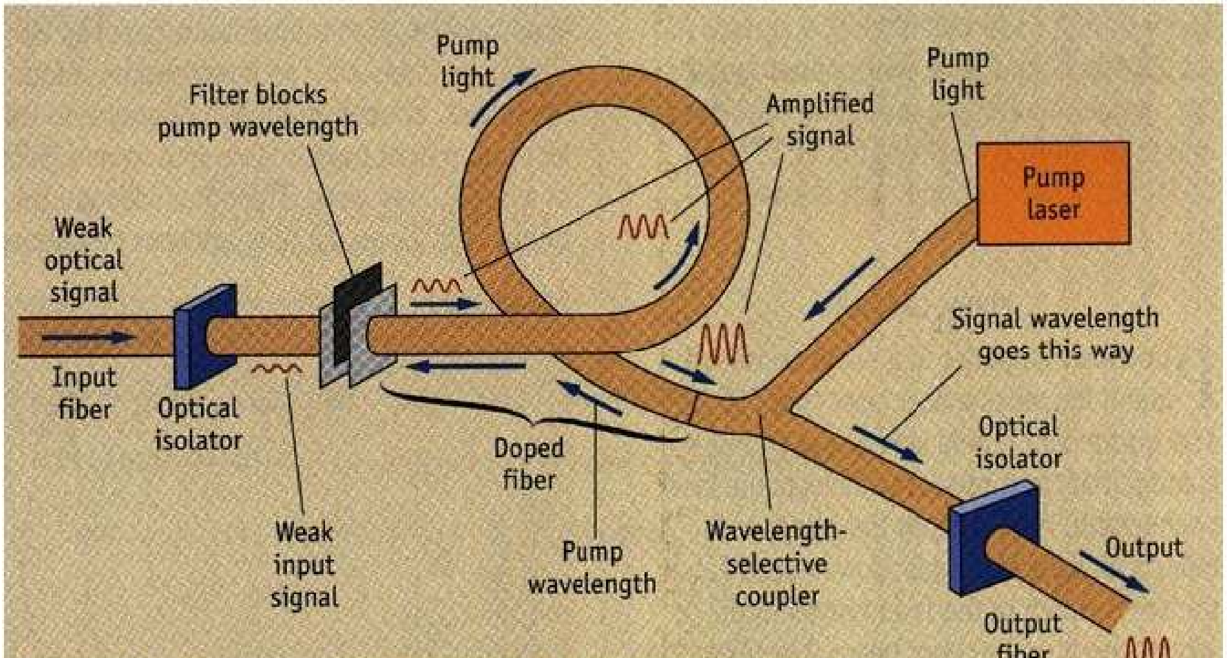
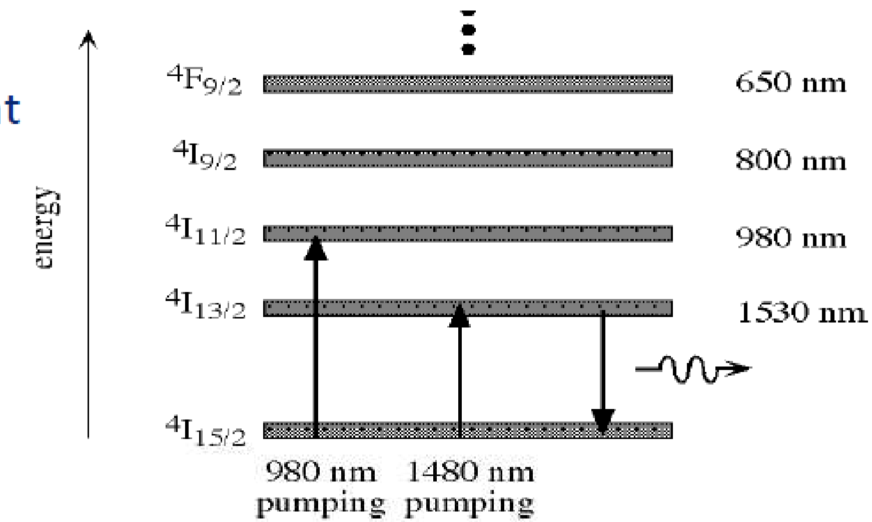
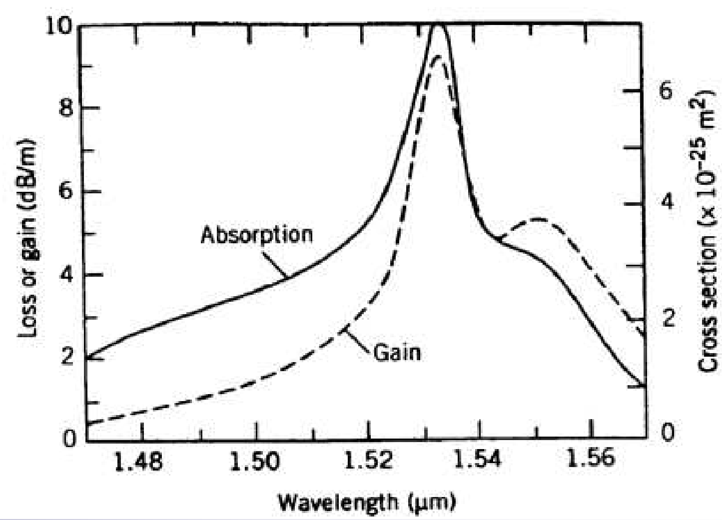


FIGURE 1. Signal wavelength is amplified as it passes through a fiber amplifier excited by a suitable pump laser. Optical isolators and pump filter block stray light from rest of the communication system. Pumping can be from either or both ends.

Pumping and gain spectrum

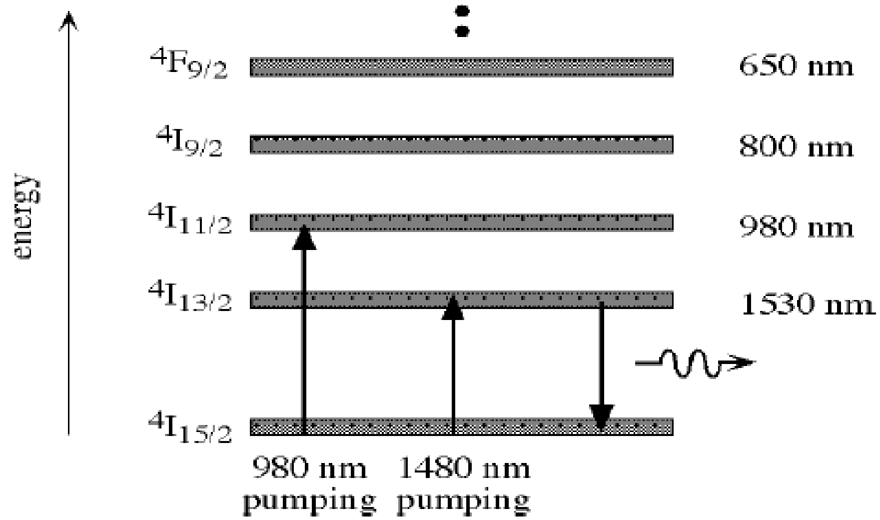
- The energy levels of Er^{3+} ions have energy levels suitable to amplify light in the 1550 nm region
  - Gain peak is at 1530 nm
  - Bandwidth is ~40 nm
- The EDFA is optically pumped at 1480 nm or 980 nm
  - 980 nm gives better performance
- Absorption and gain spectra are seen in the figure
  - Absorption is for unpumped fiber
  - Gain spectrum is shifted towards longer wavelengths





EDFA energy states

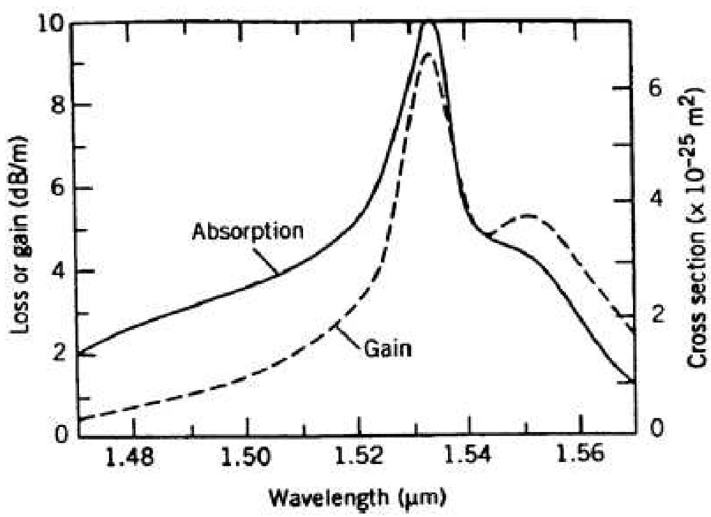
- Energy levels in Erbium are broadened into bands
  - The gain spectrum is continuous
- Typical density of Erbium in the fiber is 10^{19} ions/cm^3
  - Relative concentration of ≈ 500 ppm compared to index-raising dopants
    - Erbium is a small perturbation



- Two possible pump wavelengths:
  - 980 nm ($^4I_{15/2} - {}^4I_{11/2}$ transition), decays rapidly to $^4I_{13/2}$
  - 1480 nm ($^4I_{15/2} - {}^4I_{13/2}$ transition), pumping to edge of the first excited state
- The $^4I_{13/2}$ state is called the meta-stable state, lifetime of ≈ 10 ms
- Usually sufficient to consider only the ground state and the meta-stable state
  - The EDFA can be approximated as a two-level system

EDFA gain spectrum

- The EDFA gain spectrum depends on
  - The co-dopants (usually germanium)
  - The pump power
  - The erbium concentration
- Figure shows typical gain spectrum at large pump power and absorption spectrum (without pumping)
- The transition cross-sections describe the medium capability of producing gain and absorption
  - the EDFA cross-section is different for absorption and emission and different for the pump (σ_p^a, σ_p^e) and the signal (σ_s^a, σ_s^e)



EDFA characteristics

Advantages

- High gain (up to 50 dB possible)
- Low noise figure (3–6 dB) (noise is discussed in next lecture)
- High saturation power (> 20 dBm)
- Small coupling loss to optical fiber
- No cavity ⇒ no gain frequency dependence due to reflections
- No polarization dependence
- Does not chirp signal
- Long excited state population lifetime ⇒ no crosstalk

Disadvantages

- Not very compact (compared to a semiconductor laser)
- Operates at a fixed wavelength
- Relies on external optical pumps (not electrically pumped)

Two-level model

- The erbium population density is N_2 in the meta-stable state and N_1 in the ground state ⇒ $N_1 + N_2 = N_t$ = total erbium density
- We here assume $\sigma_p^a = \sigma_p$, $\sigma_p^e \approx 0$, $\sigma_s^a \approx \sigma_s^e = \sigma_s$, loss is negligible
- The ***rate equations*** describe the evolution of the densities

$$\frac{dN_2}{dt} = \sigma_p N_1 \Phi_p + \sigma_s (N_1 - N_2)\Phi_s - \frac{N_2}{T_1} \qquad \frac{dN_1}{dt} = -\frac{dN_2}{dt}$$

  - The ***photon fluxes*** are
    - $a_{p,s}$ are the cross-sectional area for the fiber modes

$$\Phi_p = \frac{P_p}{a_p h\nu_p} \qquad \Phi_s = \frac{P_s}{a_s h\nu_s}$$

- Signal and pump powers evolve according to
  - $\Gamma_{p,s}$ are the mode confinement factors

$$\frac{dP_s}{dz} = \sigma_s \Gamma_s (N_2 - N_1) P_s \qquad \frac{dP_p}{dz} = -\sigma_p \Gamma_p N_1 P_p$$

- This gives

$$\frac{dN_2}{dt} = -\frac{1}{\Gamma_p a_p h\nu_p}\frac{dP_p}{dz} - \frac{1}{\Gamma_p a_s h\nu_s}\frac{dP_s}{dz} - \frac{N_2}{T_1}$$

Two-level model

- A steady-state solution is obtained by setting the time derivative to zerc

$$N_2 = -\frac{T_1}{\Gamma_p a_p h\nu_p}\frac{dP_p}{dz} - \frac{T_1}{\Gamma_s a_s h\nu_s}\frac{dP_s}{dz}$$

- We obtain equations for the powers according to

$$\frac{dP_s}{dz} = \frac{\left(P_p^{'} - 1\right)\alpha_s P_s}{1 + 2P_s^{'} + P_p^{'}} \qquad \frac{dP_p}{dz} = -\frac{\left(P_s^{'} + 1\right)\alpha_p P_p}{1 + 2P_s^{'} + P_p^{'}}$$

  – where $\alpha_{p,s} = \sigma_{p,s}\Gamma_{p,s}N_t$ are pump and signal absorption coefficients and

$$P_p^{'} = \frac{P_p}{P_p^{\text{sat}}} \qquad P_s^{'} = \frac{P_s}{P_s^{\text{sat}}} \qquad P_{p,s}^{\text{sat}} = \frac{a_{p,s} h\nu_{p,s}}{\sigma_{p,s} T_1}$$

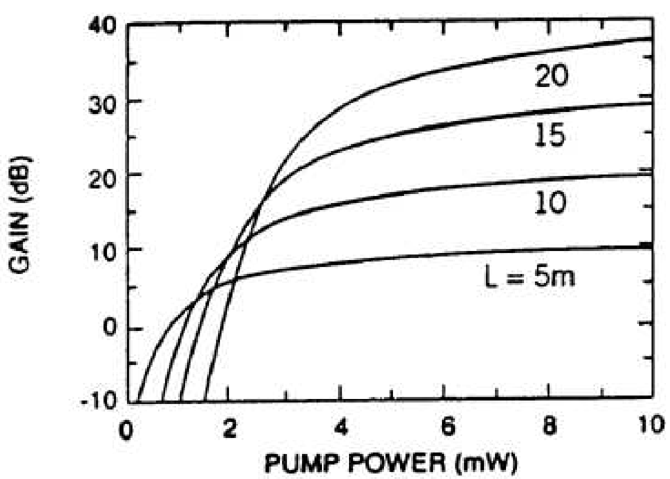
- Use N_1 and N_2 solutions in P_s and P_p eqs on last slide, integrate $z = 0$ to L

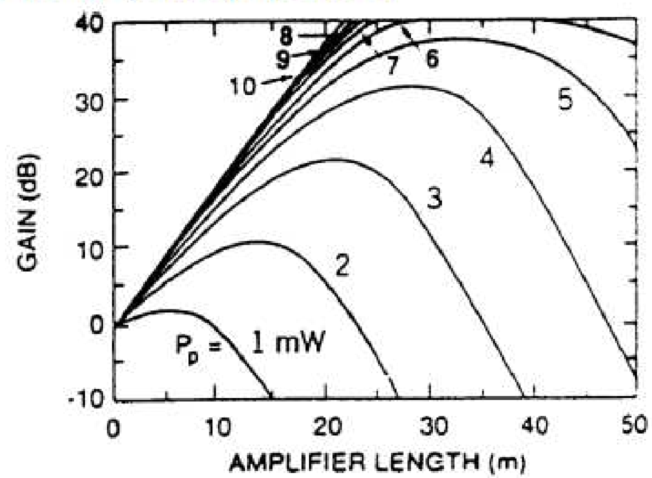
$$\frac{P_p(L)}{P_p(0)} = e^{-\alpha_p L}\exp\left[\frac{P_p(0) - P_p(L)}{P_p^{\text{sat}}} + \frac{\Gamma_p \nu_p a_p}{\Gamma_s \nu_s a_s}\frac{P_s(0) - P_s(L)}{P_p^{\text{sat}}}\right]$$

$$\frac{P_s(L)}{P_s(0)} = e^{-\alpha_s L}\exp\left[\frac{P_s(0) - P_s(L)}{P_s^{\text{sat}}/2} + \frac{\Gamma_s \nu_s a_s}{\Gamma_p \nu_p a_p}\frac{P_p(0) - P_p(L)}{P_s^{\text{sat}}/2}\right]$$

EDFA gain modeling results

- The (implicit) analytical expressions can be used to study the EDFA gain
- Pump power increase ⇒ small-signal gain increases
  - Until all ions are excited, full population inversion, gives lowest noise figure
- Longer EDFA ⇒ more ions to excite ⇒ (potentially) larger gain
- For fixed pump power, an optimal length exists that maximizes the gain
  - shorter fiber ⇒ the pump power is not fully used
  - too long fiber ⇒ part of EDFA is not sufficiently pumped
- 35 dB gain can be realized with < 10 mW pump power





System aspects of EDFAs

Multi-channel amplification in EDFAs:

- $T_{1,EDFA} \approx 10$ ms, the amplifier is “slow” to react on changing input power
  - No problems related to gain modulation when doing WDM amplification

Accumulation of ASE:

- In cascaded EDFAs, ASE will cause two particular problems
  - Increasing degradation of the SNR after each amplifier
  - Eventually gain saturation caused by the ASE ⇒ less signal gain

Pulse amplification in EDFAs:

- Amplification of pulses in saturated EDFAs do not suffer from chirp or distortion due to gain dynamics
- For very short pulses (< 1 ps) however:
  - Gain is reduced in the spectral wings due to the finite bandwidth
  - GVD and nonlinearities will influence the pulse (EDFA length ~ 100 m)

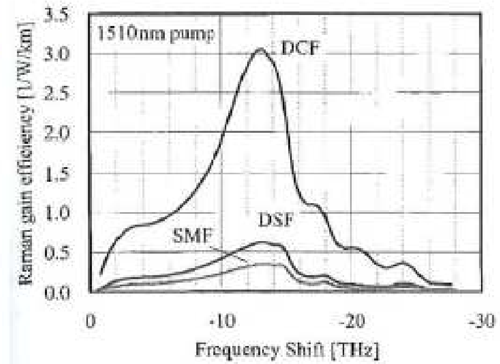
Raman amplifiers

- Raman amplifiers are based on ***stimulated Raman scattering***
- The pump and the signal co-propagate
  - Power is transferred during transmission
  - The pump wavelength is shorter than the signal wavelength
  - The excess energy is given to the medium (the fiber)
    - A molecular vibration (optical phonon) is created
- The pump and signal can propagate in different directions

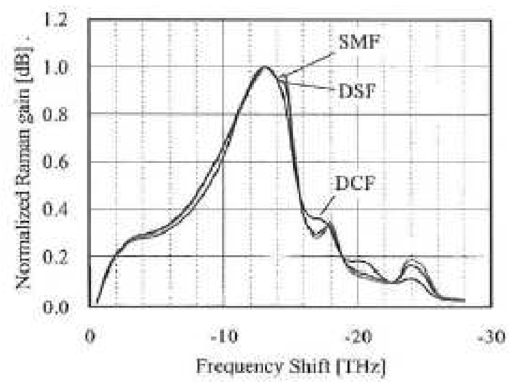
Raman gain and bandwidth

- The Raman gain spectrum is a property of amorphous glass
- The Raman gain coefficient is proportional to the pump intensity I_p

$$g(\omega, z) = g_R(\omega) I_p(z) = \frac{g_R(\omega)}{a_p} P_p(z)$$



  - a_p is the cross-sectional area of the pump beam, depends on fiber type
    - DCF has a small core diameter and a large g_R/a_p
    - Figure shows g_R/a_p and normalized gains
- Gain peaks at 13.2 THz
  - Shift between absorbed and emitted photons is called ***Stokes shift***
    - Similar gain spectra for all fibers
  - The FWHM of the gain peak is nearly 6 THz



- Requires rather high powers
  - Example (see the book): G > 20 dB requires > 5 W in a 1-km-long fiber

Raman induced signal gain

- Consider a CW signal and a CW pump
  - Frequency ratio occurs since pump photons have higher power

$$dP_s / dz = -\alpha_s P_s + (g_R / a_p) P_p P_s$$

$$dP_p / dz = -\alpha_p P_p - (\omega_p / \omega_s)(g_R / a_p) P_s P_p$$

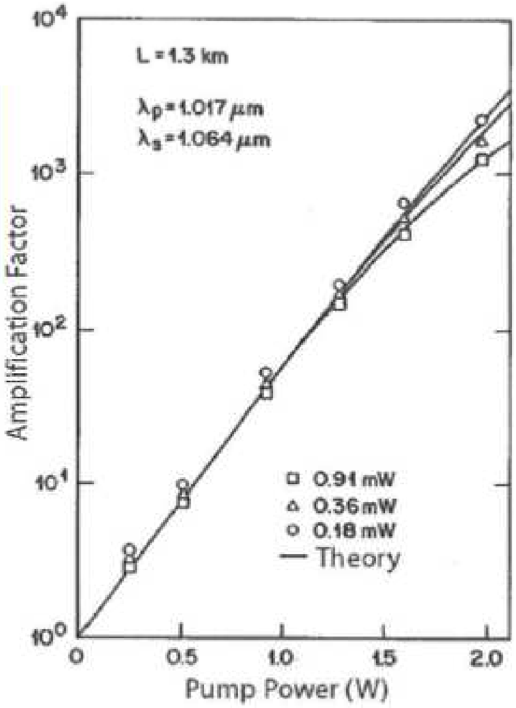
- If the pump is undepleted (affected only by loss), we have

$$P_s(L) = P_s(0)\exp[(g_R / a_p) P_p(0) L_{\text{eff}} - \alpha_s L] \qquad L_{\text{eff}} = [1 - \exp(-\alpha_p L)] / \alpha_p$$

- The amplifier gain is given by

$$G_A = \exp(g_0 L), \;\; g_0 = g_R \left(\frac{P_0}{a_p}\right)\left(\frac{L_{\text{eff}}}{L}\right) \approx \{\alpha_p L >> 1\} \approx \frac{g_R P_0}{a_p \alpha_p L}$$

  - Obtained as (output power with Raman)/(output power without Raman)
- Figure shows:
  - Amplifier gain increases exponentially...
  - ...until gain saturation occurs

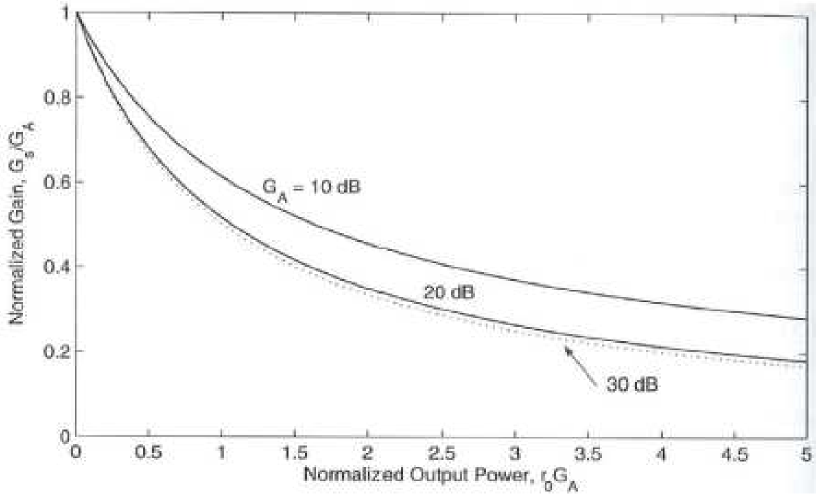


Raman induced signal gain

- The pump supplies the signal with power
  - When the transferred energy is significant, the pump is ***depleted***
  - The gain is decreased, referred to as ***gain saturation***
- The saturated amplified gain can be found numerically...
  - Like in figure on previous slide
- ...or analytically, assuming $\alpha_s = \alpha_p$, as

$$G_s = \frac{(1+r_0)\exp(-\alpha_s L)}{r_0 + G_A^{-(1+r_0)}}, \quad r_0 = \frac{\omega_p}{\omega_s}\frac{P_s(0)}{P_p(0)}$$

- Figure shows gain-saturation characteristics
  - Gain is reduced by 3 dB when $G_A r_0 \approx 1$
  - In a system, Raman amplifiers typically operate in the unsaturated regime



Multiple-pump Raman amplification

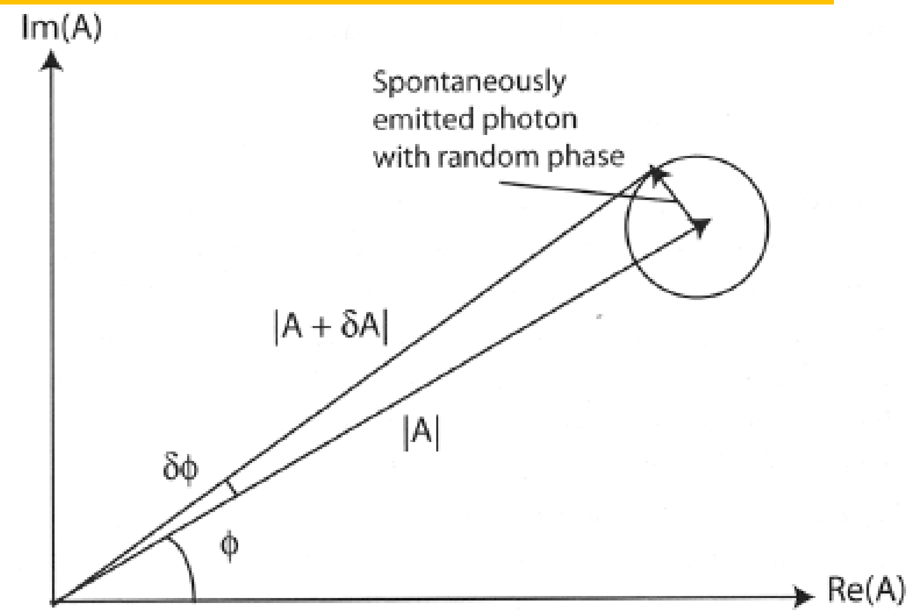
- A WDM system with many channels require broadband amplifiers
  - 100 channels may require uniform gain over 70–80 nm (~10 THz)
- Broadband amplification can be achieved with
  - Hybrid EDFA/Raman amplification
  - Raman amplification using multiple pump lasers
- Multiple-pump Raman amplification
  - Will set up a superposition of gain spectra
    - Is more broadband since it is flatter
  - Requires careful selection of the pump wavelengths
    - Will determine the gain ripples
  - Requires consideration of pump–pump interactions
    - The pump waves are affected by the Raman interaction
    - In general, this is studied numerically
      - A large coupled system of differential equations must be solved

Noise in optical fiber

- Noise from optical amplifiers
  - EDFA noise
  - Raman noise
- Optical SNR (OSNR), noise figure, (electrical) SNR
- Amplifier and receiver noise
  - ASE and shot/thermal noise
- Preamplification for SNR improvement

Amplifier noise

- All amplifiers add noise
  - To amplify (make a larger copy), a physical device must "observe" the signal
  - Cannot be done without perturbing the signal
    - Assured by the Heisenberg uncertainty principle
- Lumped and distributed amplification have different performance
- Noise comes from spontaneously emitted photons
- These have random
  - direction
  - polarization
  - frequency (within the band)
  - phase
- Some of these add to the signal
  - Causes intensity and phase noise

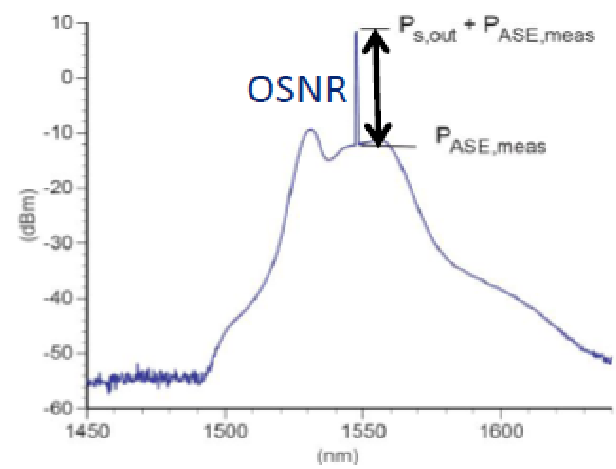


Definition of the optical SNR

- Optical signals are often characterized by the ***optical SNR*** (OSNR)
  - Easily measured with an ***optical spectrum analyzer*** (OSA)
    - Makes signal monitoring in the lab easy ⇒ is very popular
- The definition of the OSNR is

$$\mathrm{OSNR} = \frac{P_{\mathrm{signal X}} + P_{\mathrm{signal Y}}}{P_{\mathrm{noise X}} + P_{\mathrm{noise Y}}} = \{\text{For single polarization signal}\} = \frac{P_{\mathrm{signal}}}{2P_{\mathrm{ASE}}}$$

  - The index X and Y denote the two polarizations
- The OSNR is related to the SNR, *Q*, and BER
- OSNR is usually normalized to a 0.1 nm bandwidth
  - Entire signal power is included, noise is measured over 0.1 nm
  - Implies required OSNR (for given BER) is bit rate-dependent



EDFA noise

- The noise is called ***amplified spontaneous emission*** (ASE)
  - Is being amplified since there is gain
  - Will reach the receiver (remaining optical path is amplified)
- The ASE power at the output of the EDFA

$$P_{\mathrm{ASE}} = S_{\mathrm{ASE}} \Delta \nu_o = n_{\mathrm{sp}} h \nu_0 (G-1) \Delta \nu_o$$

  - $\Delta\nu_0$ is the effective bandwidth of the optical filter used to suppress noise
  - S_{ASE} is the (onesided) noise power spectral density (PSD)
  - This is the power per polarization
- n_{sp} is the ***spontaneous-emission factor*** also known as the ***population-inversion factor***
  - For an EDFA

$$n_{\mathrm{sp}} = \frac{\sigma_s^e N_2}{\sigma_s^e N_2 - \sigma_s^a N_1} \approx \frac{N_2}{N_2 - N_1} > 1$$

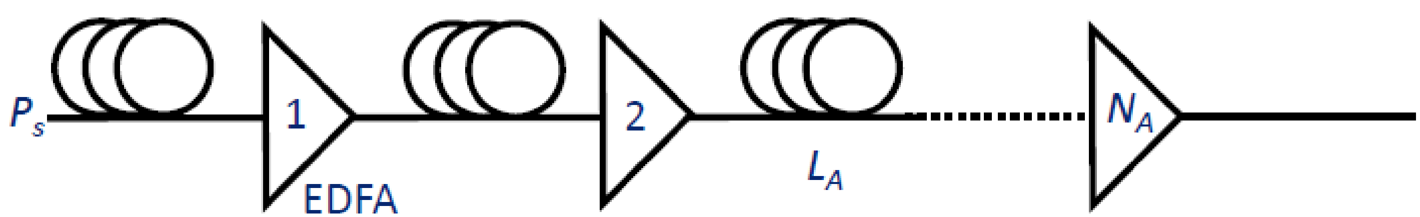
OSNR due to EDFA noise

- The OSNR is reduced each time a signal is amplified
  - Each EDFA add to the noise PSD due to the generation of more ASE
- After N_A amplifiers in a link with span loss equal to the gain in each amplifier and with identical EDFA noise performance, we have

$$\mathrm{OSNR} = \frac{P_{\mathrm{in}}}{N_A 2P_{\mathrm{ASE}}} = \frac{P_{\mathrm{in}}}{N_A 2n_{\mathrm{sp}} h\nu_o \Delta\nu_o (G-1)} \approx \frac{P_{\mathrm{in}}}{N_A 2n_{\mathrm{sp}} h\nu_o \Delta\nu_{0.1} G}$$

- In dB and dBm at 1550 nm and $\Delta\nu_0 = 0.1$ nm, we have

$$\mathrm{OSNR}_{\mathrm{dB}} = P_{\mathrm{in}}[\mathrm{dBm}] - N_A[\mathrm{dB}] - 2n_{\mathrm{sp}}[\mathrm{dB}] - G[\mathrm{dB}] + 58\,\mathrm{dBm}$$



OSNR due to EDFA noise, example

What is the max. transmission distance with 100 km or 50 km EDFA spacing?

- A 10 Gbit/s system with a OSNR requirement of 20 dB
- The loss is 0.25 dB/km and $2n_{sp}$ = 5 dB
- The launched power into each span is 1 mW per WDM channel

$$\mathrm{OSNR}_{\mathrm{dB}} = P_{\mathrm{in}}[\mathrm{dBm}] - N_A[\mathrm{dB}] - 2n_{\mathrm{sp}}[\mathrm{dB}] - G[\mathrm{dB}] + 58\,\mathrm{dBm}$$

- L_A = 100 km ⇒ N_A = 8 dB = 6.3 ⇒ 6 amps ⇒ 700 km
- L_A = 50 km ⇒ N_A= 20.5 dB = 112.2 ⇒ 112 amps ⇒ 5650 km

- The amplifier spacing plays a critical role for the OSNR
  - Short L_A: Noise accumulates slowly ⇒ high OSNR at receiver
  - Long L_A: Few EDFAs are needed ⇒ system cost is lower
- Shows trade-off between cost and performance
  - Techniques that enable cost reduction are desirable
    - This can, for example, be error correction or distributed amplification
- Hints that distributed amplification may perform better

OSNR due to EDFA noise, amplifier spacing

- We can express the number of amplifiers as
  - L_T is the total system length
  - This gives the OSNR

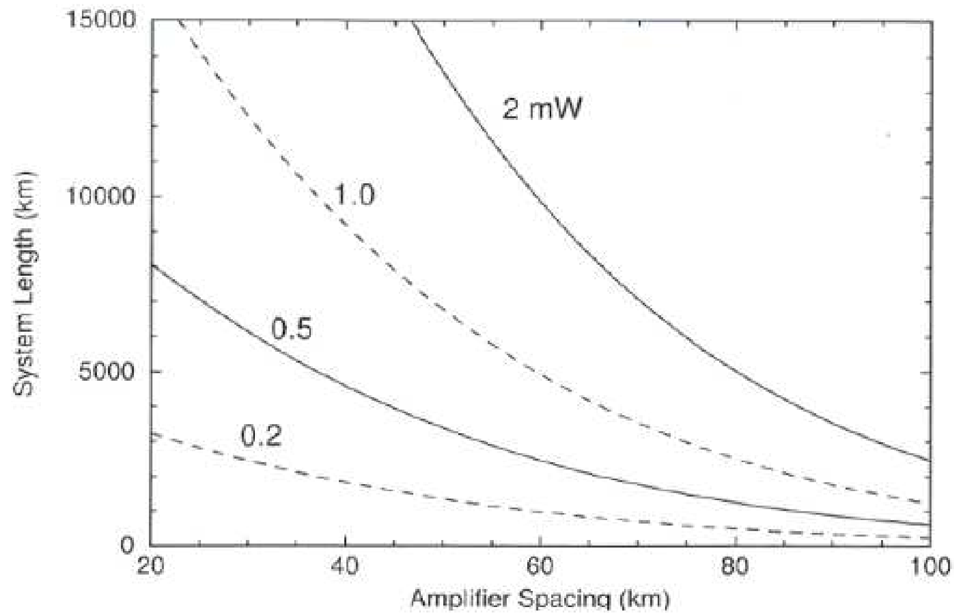
$$N_A = \frac{\alpha L_T}{\ln G}$$

$$\mathrm{OSNR} = \frac{P_{\mathrm{in}} \ln G}{2 n_{\mathrm{sp}} h \nu_0 \Delta \nu_{0.1} \alpha L_T (G-1)}$$

- We see that

$$P_{\mathrm{ASE}} \propto \frac{G-1}{\ln G}$$

- Figure shows maximum system length = "system reach"
  - OSNR = 20 dB
  - α = 0.2 dB/km
  - n_{sp}=1.6
  - $\Delta\nu_0$ = 100 GHz



Raman amplifier noise

- Noise is generated by ***spontaneous Raman scattering***
- The noise PSD per polarization after an amplified fiber is

$$S_{ASE} = n_{sp} h\nu_0 G(L) \int_0^L \frac{g_0(z)}{G(z)} dz \qquad G(z) = \exp\left(\int_0^z [g_0(\zeta) - \alpha_s] d\zeta \right) \qquad g_0(z) = \frac{g_R P_p(z)}{a_p}$$

  - Depends on the net power gain, $G(L)$
    - Observe: This is ***net*** gain, for a transparent system $G(L) = 1$
  - Depends on the distribution of gain $g_0(z)$
- n_{sp} has a different definition for Raman amplification
  - h is Planck's constant
  - ν_R is the Raman shift
    - Maximum gain at 13.2 THz
  - k_B is Boltzmann's constant
  - T is the temperature, ≈ 293 K

$$n_{sp} = \frac{1}{1 - \exp(-h\nu_R / k_B T)}$$

- This gives $n_{sp} = 1.13$, $n_{sp} \to 1$ as $T \to 0$

Raman amplifier noise, example

- The pump experiences loss ⇒ gain is not constant
  - Anyway, as an example, study an amplified *transparent* fiber, $g_0 = \alpha_s$
  - We then have...

$$G(z) = \exp\left(\int_0^z [g_0(\zeta) - \alpha_s] d\zeta \right) = \exp\left(\int_0^z 0\, d\zeta \right) = 1$$

...and the noise PSD becomes

$$S_{\mathrm{ASE}} = n_{\mathrm{sp}} h \nu_0 G(L) \int_0^L \frac{g_0(z)}{G(z)} dz = n_{\mathrm{sp}} h \nu_0 \int_0^L \alpha_s dz = n_{\mathrm{sp}} h \nu_0 \alpha_s L$$

  - We compare this with the case where an EDFA is placed at the end

$$S_{\mathrm{ASE}} = n_{\mathrm{sp}} h \nu_0 (G-1) = n_{\mathrm{sp}} h \nu_0 (e^{\alpha_s L} - 1)$$

  - n_{sp} is similar in both cases (somewhat better for Raman)
  - The final terms are very different, study $\exp(\alpha_s L)$ = 20 dB

$$\alpha_s L \approx 4.6, (e^{\alpha_s L} - 1) = 99$$

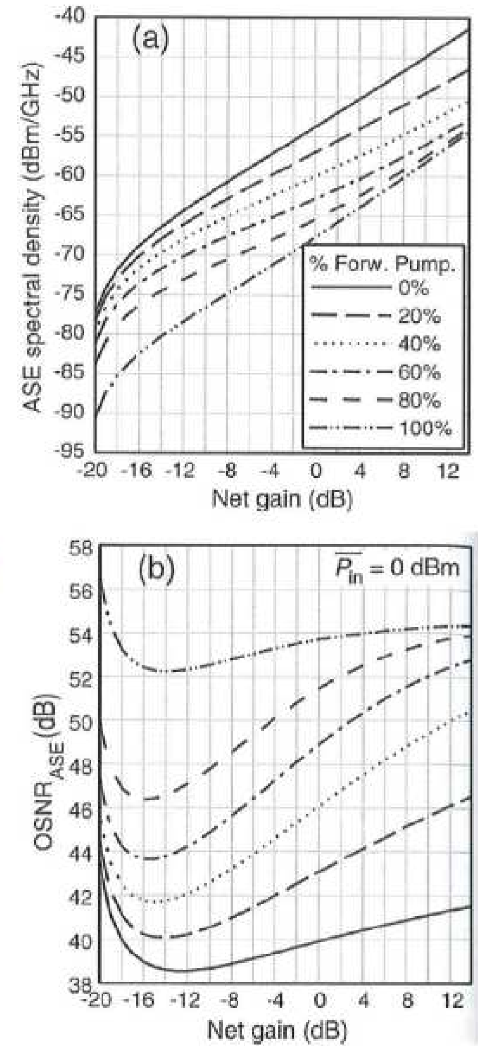
Distributed amplification can be vastly superior to lumped amplification

OSNR due to Raman noise

- Pump stations are set up spaced by L_A
  - Gain is designed to make $P_s(z = nL_A) = P_{in}$
- The OSNR is given by

$$\mathrm{OSNR} = \frac{P_{in}}{2N_A S_{ASE} \Delta\nu_{0.1}}$$

  - S_{ASE} must be found using the general expression
  - Depends on pumping; forward, backward, or both
- Figure shows ASE PSD and OSNR, fiber is 100 km long
  - Pumping is bidirectional to varying degree
  - System is transparent at 0 dB net gain
  - Forward pumping is better than backward pumping
    - Nonlinearities are not considered

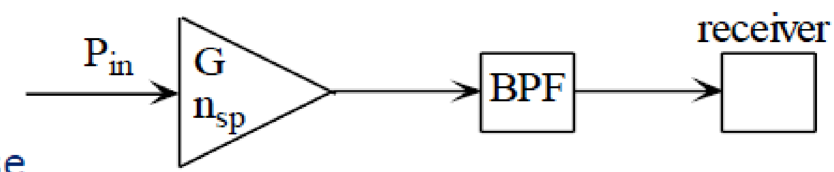


Raman amplifier performance

- In general, it is preferable to amplify a strong signal
  - For a given gain (and added noise PSD), the (O)SNR decrease is smaller
  - Forward pumping is better than backward pumping
  - Unfortunately, signal power must be limited due to nonlinearities
- The Raman amplifier is affected by several phenomena:
  - Double Rayleigh scattering occurs
    - Light scattered back is scattered again
  - Pump-noise transfer decreases the SNR
    - The gain changes with the pump intensity fluctuations
  - The amplifier is polarization dependent
    - Is counteracted using polarization scrambling

Electrical signal-to-noise ratio (SNR)

- The Q and BER are determined by the SNR in the detected current
  - Agrawal calls this ”electrical signal-to-noise ratio” to separate from OSNR
- An EDFA can improve the sensitivity of a thermally noise limited receiver
  - A ***preamplified optical receiver***
  - The added optical noise can be much smaller than the thermal noise



- The generated photocurrent in the receiver is
  - E_{cp} = ASE co-polarized with signal
  - E_{op} = ASE orthogonal with signal
  - i_s = Shot noise
  - i_T = Thermal noise

$$I = R_d\left(\left|\sqrt{G}E_s + E_{cp}\right|^2 + \left|E_{op}\right|^2\right) + i_s + i_T$$

- The ASE has a broad spectrum, and can be written
  - The magnitude square is a multiplication ⇒ new frequencies are generated ⇒ “***beating***”

$$E_{cp} = \sum_{m=1}^{M} (S_{ASE}\Delta\nu_s)^{1/2} \exp(i\phi_m - i\omega_m t)$$

Electrical signal-to-noise ratio (SNR)

- The received electrical current is
  - $i_{\text{sig-sp}}$ = signal-ASE beat noise term
  - $i_{\text{sp-sp}}$ = ASE-ASE beat noise term

$$I = R_d G P_s + i_{\text{sig-sp}} + i_{\text{sp-sp}} + i_s + i_T$$

- The variance of the noise terms are

$$\sigma^2_{\text{sig-sp}} = 4R_d^2 G P_s S_{\text{ASE}} \Delta f \qquad \sigma^2_{\text{sp-sp}} = 4R_d^2 S_{\text{ASE}}^2 \Delta f (\Delta \nu_0 - \Delta f / 2)$$

$$\sigma_s^2 = 2q[R_d (G P_s + P_{\text{ASE}})]\Delta f \qquad \sigma_T^2 = (4 k_B T / R_L)\Delta f$$

  - $\Delta\nu_0$ is bandwidth of optical bandpass filter (rejects out-of-band noise)

- The SNR is here defined as

$$\text{SNR} = \frac{\langle I \rangle^2}{\sigma^2} = \frac{(R_d G P_s)^2}{\sigma^2_{\text{sig-sp}} + \sigma^2_{\text{sp-sp}} + \sigma_s^2 + \sigma_T^2}$$

Impact of ASE on SNR

- Let us compare the SNR without and with amplification by an EDFA
  - Amplifier and bandpass filter is inserted before the receiver

$$\mathrm{SNR}_{\mathrm{no\,amp}} = \frac{(R_d P_s)^2}{\sigma_s^2 + \sigma_T^2}, \quad \mathrm{SNR}_{\mathrm{amp}} = \frac{(R_d G P_s)^2}{\sigma_{\mathrm{sig-sp}}^2 + \sigma_{\mathrm{sp-sp}}^2 + \sigma_s^2 + \sigma_T^2}$$

  - Notice that σ_s are different in the two cases (σ_T stays the same)
  - We neglect $\sigma_{\mathrm{sp-sp}}$ and the noise current contribution to shot noise to get

$$\frac{\mathrm{SNR}_{\mathrm{amp}}}{\mathrm{SNR}_{\mathrm{no\,amp}}} = \frac{(R_d G P_s)^2}{(4R_d^2 G P_s S_{\mathrm{ASE}} \Delta f) + (2qR_d G P_s \Delta f) + \sigma_T^2} \frac{(2qR_d P_s \Delta f) + \sigma_T^2}{(R_d P_s)^2}$$

  - We use the PSD and the ideal responsivity

$$S_{\mathrm{ASE}} = n_{\mathrm{sp}} h\nu_0 (G-1) \approx \{G >> 1\} \approx n_{\mathrm{sp}} h\nu_0 G \qquad R_d = q/(h\nu_0)$$

  - We get

$$\frac{\mathrm{SNR}_{\mathrm{amp}}}{\mathrm{SNR}_{\mathrm{no\,amp}}} = \frac{1 + k_T}{2n_{\mathrm{sp}} + 1/G + k_T/G^2} \qquad k_T = \frac{\sigma_T^2}{2qR_d P_s \Delta f}$$

    - Notice: k_T is ratio (thermal noise)/(shot noise) without amplification
    - All quantities in the denominator ($2qR_dP_s\Delta f$) are kept constant!

A thermal noise-limited receiver

- How is the SNR changed in the thermal limit?
- First assume that thermal noise dominates *before and after* amplification

$$\frac{\mathrm{SNR}_{\mathrm{amp}}}{\mathrm{SNR}_{\mathrm{no\,amp}}} = \frac{1+k_T}{2n_{\mathrm{sp}}+1/G+k_T/G^2} \approx \frac{k_T}{k_T/G^2} = G^2$$

  - There is a huge improvement in the SNR
    - Signal power is increased, noise power remains constant
- However, at high *G*, we cannot ignore the other noise terms
- Study the realistic case that thermal noise dominates before and is negligible after amplification

$$\frac{\mathrm{SNR}_{\mathrm{amp}}}{\mathrm{SNR}_{\mathrm{no\,amp}}} = \frac{1+k_T}{2n_{\mathrm{sp}}+1/G+k_T/G^2} \approx \frac{k_T}{2n_{\mathrm{sp}}+1/G} \approx \frac{k_T}{2n_{\mathrm{sp}}}$$

  - SNR improvement saturates as *G* is increased
  - Improvement can be very large

In the thermal limit, amplification improves the SNR

A shot noise-limited receiver, noise figure

- Now instead assume that the optical signal has high power
  - Thermal noise is negligible

$$\frac{\mathrm{SNR}_{\mathrm{amp}}}{\mathrm{SNR}_{\mathrm{no\,amp}}} = \frac{1+k_T}{2n_{\mathrm{sp}} + 1/G + k_T/G^2} \approx \frac{1}{2n_{\mathrm{sp}} + 1/G} \approx \frac{1}{2n_{\mathrm{sp}}}$$

  - The SNR is decreased by the amplification

EDFA amplification of a perfect signal decreases the SNR by > $2n_{\mathrm{sp}}$ (> 3dB)

- The ***noise figure*** is defined

$$\mathrm{NF} = F_n \equiv \frac{(\mathrm{SNR})_{\mathrm{in}}}{(\mathrm{SNR})_{\mathrm{out}}}$$

- The SNR values are what you would obtain by putting an ideal receiver before and after an EDFA, respectively
  - Ideal means shot noise-limited, 100% quantum efficiency
- Our study above has provided us with the (inverse) minimum value

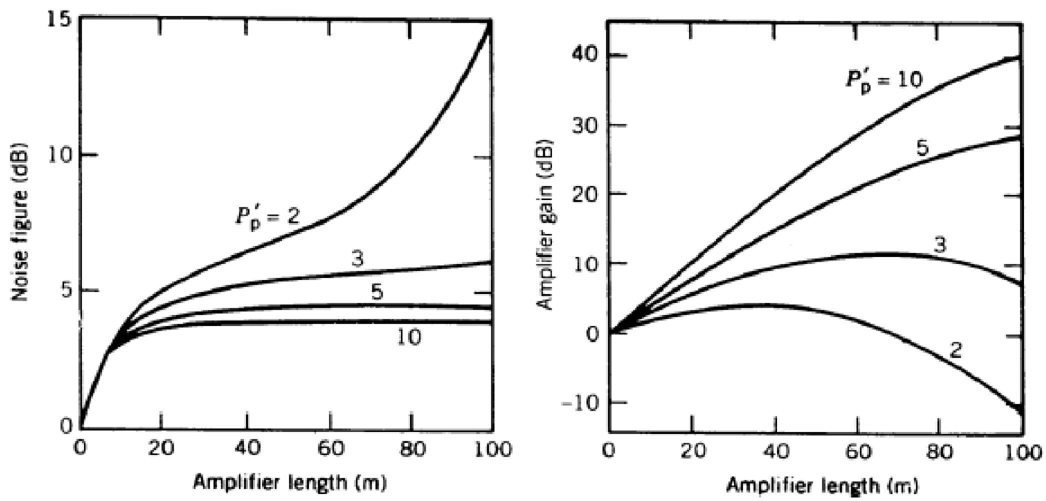
$$F_n \approx 2n_{\mathrm{sp}} \geq 2$$

Noise figure

- For an EDFA, the noise figure is

$$F_n \approx 2n_{sp} \qquad n_{sp} = \frac{\sigma_s^e N_2}{\sigma_s^e N_2 - \sigma_s^a N_1} \approx \frac{N_2}{N_2 - N_1} > 1$$

- In reality, N_1 and N_2 change along the EDFA
  - Pump power and signal power are not constant
  - The rate equations can be solved numerically
- Figure shows
  - Noise figure and amplifier gain as a function of...
  - ...pump power and amplifier length
- A long amplifier
  - Can provide high gain
  - Requires high pump power



Noise figure

- The noise figure is increased
  - If the population inversion is incomplete (somewhere in the amplifier)
  - If there are coupling losses into the amplifier
- Pumping is facilitated by pumping at 980 nm
  - No stimulated emission caused by pump photons ($\sigma_p^e \approx 0$)
    - Corresponding energy level is almost empty (short-lived)
  - Noise figure ≈ 3 dB is possible, 3.2 dB has been measured
- With 1480 nm pumping $\sigma_p^e \neq 0$
  - Ground state will always be populated by some ions
    - Some excited ions will be stimulated by pump photons to relax
  - Noise figure is larger for this case
- Coupling into and out of an EDFA is efficient
- Typical EDFA modules have F_n = 4–6 dB

SNR/OSNR relation

- In general, there is no simple relation between the OSNR and the SNR
  - OSNR is prop. to the optical power, SNR is prop. to the electrical power
  - Electrical power is proportional to the (optical power)^2
    - Not true in a coherent receiver
- When signal–ASE noise is dominating we have

$$\mathrm{SNR} \approx \frac{(R_d G P_s)^2}{4R_d^2 G P_s S_{\mathrm{ASE}} \Delta f} = \frac{G P_s \Delta\nu_{0.1}}{4P_{\mathrm{ASE}} \Delta f} = \frac{\Delta\nu_{0.1}}{2\Delta f}\mathrm{OSNR}$$

- For a single-polarization signal, we can use

$$\mathrm{OSNR} = \frac{P_s}{2S_{\mathrm{ASE}}\Delta\nu_{0.1}} = \frac{E_s}{S_{\mathrm{ASE}}}\frac{f_s}{2\Delta\nu_{0.1}}$$

  - E_s is the energy per symbol, f_s is the symbol rate (in baud)
  - E_s/S_{ASE} is often written E_s/N_0 is digital communication literature
  - The relation between E_s/N_0 and the BER depends on the type of receiver, modulation format and more

Receiver sensitivity and Q factor

- When shot noise and thermal noise are negligible:
  - The statistics are not Gaussian (cannot have negative current)...
  - ...but Gaussian statistics are often used anyway for simplicity

$$\sigma_1^2 = \sigma_{\text{sig-sp}}^2 + \sigma_{\text{sp-sp}}^2 + \sigma_s^2 + \sigma_T^2 \approx \sigma_{\text{sig-sp}}^2 + \sigma_{\text{sp-sp}}^2 \qquad \sigma_0^2 = \sigma_{\text{sp-sp}}^2 + \sigma_T^2 \approx \sigma_{\text{sp-sp}}^2$$

- The receiver sensitivity is then

$$\bar{P}_{\text{rec}} = h\nu_0 F_o \Delta f \left(Q^2 + Q\sqrt{\frac{\Delta\nu_0}{\Delta f} - \frac{1}{2}} \right)$$

- Assuming that $P_{\text{rec}} = N_p h\nu_0 B$ and $\Delta f = B/2$, we get

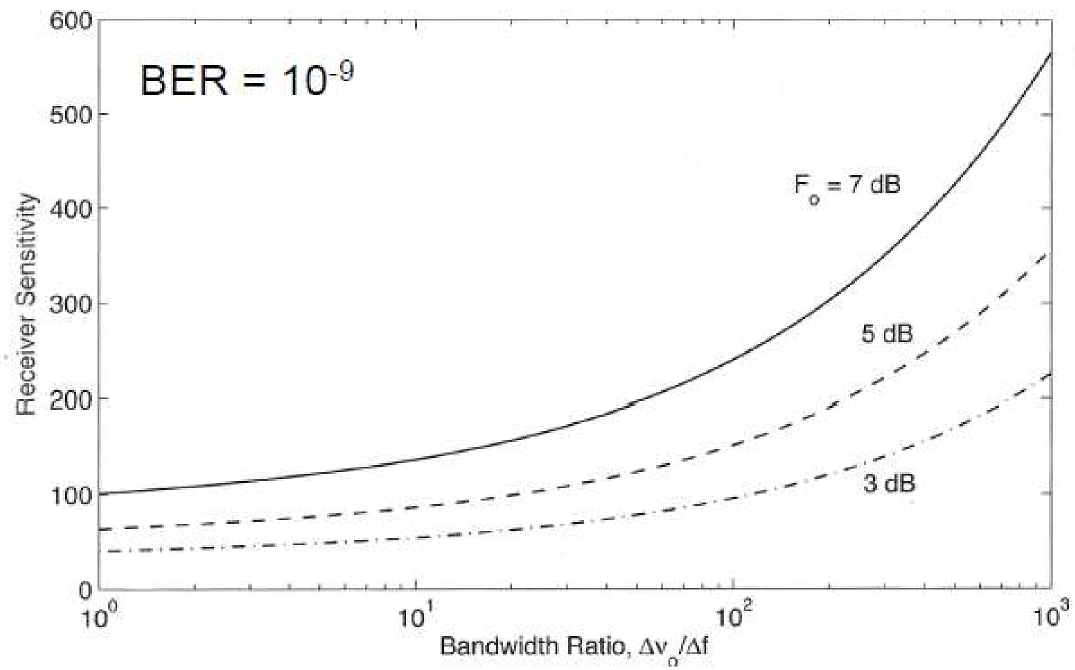
$$\bar{N}_p = \frac{1}{2} F_o \left(Q^2 + Q\sqrt{\frac{\Delta\nu_0}{\Delta f} - \frac{1}{2}} \right)$$

- The number of photons per bit depends on
  - The BER (via Q), the noise figure, and the receiver bandpass filter

Low-noise amplification and narrow filtering is critical for high performance

Receiver sensitivity of preamplified receiver

- Using F_o = 2, Q = 6, $\Delta\nu_0 = B \Rightarrow N_p$= 43 photons per bit on average
- The quantum limit is N_p = 10 photons per bit on average



N_p = 100 is realistic with a reasonable noise figure and filter bandwidth

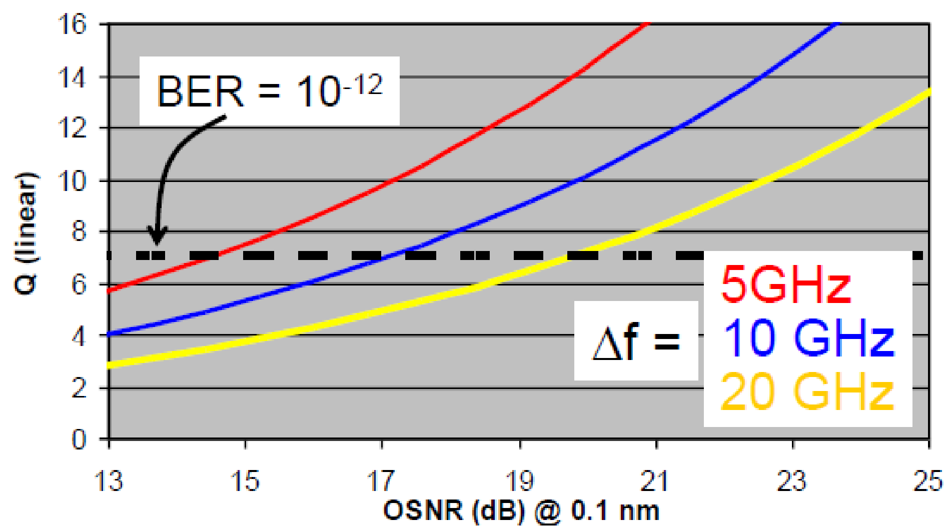
Relation between Q and the OSNR

- When ASE noise dominates, we have
  - $\Delta\nu_o$ = bandwidth of optical bandpass filter [nm]
  - Δf = equivalent receiver electrical bandwidth [GHz]

$$Q = \sqrt{125\frac{\Delta\nu_o}{\Delta f}}\,\frac{2OSNR\cdot\frac{0.1}{\Delta\nu_o}}{\sqrt{4OSNR\cdot\frac{0.1}{\Delta\nu_o}+1}+1}$$

If we know the OSNR and the bandwidths, we can find *Q* and the BER

- In figure, $\Delta\nu_o$ = 0.4 nm
  - Reasonable value for a 10 Gbit/s system
- The necessary OSNR = 15–20 dB at a bit rate of 10 Gbit/s



Optimum launched power

- Amplifiers cancel the loss, but noise and nonlinearities are accumulated
  - High power ⇒ potentially higher SNR but also more nonlinear distortion
  - As power is increased, BER first drops, then increases again

There is an optimal launch power that minimizes the BER